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# A Nonstationary Generalization of the Kerr–Newman Metric

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A new metric depending on three arbitrary parameters is presented by the method of complex coordinate transformations. It gives the gravitational field of a radiating rotating charged body. The metric is algebraically special of Petrov type II according to classification of the Weyl tensor, with a twisting, shear-free, null congruence identical to that of the Kerr–Newman metric. The new metric bears the same relation to the Kerr–Newman metric as the Bonner–Vaidya metric does to the Reissner–Nordstrom metric.

## 1. INTRODUCTION

The Schwarzschild (Reissner–Nordstrom) metric and Kerr (Kerr–Newman) metric describing the exterior gravitational field of a spherically symmetric (charged) body and a rotating (charged) body, respectively, are very fundamental in general relativity theory and have numerous applications in experimental relativity and astrophysics. However, as is well known, in general, celestial bodies have radiation. Hence, the gravitational fields surrounding spherically symmetric (charged) bodies and rotating (charged) bodies cannot be described by such metrics except in the approximation in which one neglects the energy density of the emitted radiation. Such an approximation may not be valid for certain astrophysical processes, in which case the radiation must be taken into account. Thus, much attention has focused on finding metrics which describe the external gravitational field of radiating bodies. Vaidya (1943, 1951, 1953) and Bonner and Vaidya (1970) obtained a radiating metric and a radiating charged metric, respectively, which are nonstatic generalizations of the Schwarzschild metric and Reissner–

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Nordstrom metric, respectively. Carmeli (1982; Carmeli and Kaye, 1977, 1979) generalized the Kerr metric and got the metric of a radiating rotating body, which bears the same relation to the Kerr metric as does Vaidya's metric to the Schwarzschild metric. However, to the best of our knowledge, a nonstatic generalization of the Kerr–Newman metric has not yet been derived. In this paper we present and discuss the metric outside a rotating charged body when radiation is included.

A generation method using "complex tricks," i.e., a complex coordinate transformation, was first used by Newman and Janis (1965a,b) and justified and further developed by several authors (Talbot, 1969; Newman, 1973; Schiffer *et al.*, 1973; Jing *et al.*, 1992). Newman (1973; Newman and Janis, 1965a,b) pointed out that such an operation works for solutions of the form $g_{\mu\nu} = \eta_{\mu\nu} + Hl_\mu l_\nu$. Talbot (1969) justified such a method within the formalism for algebraically special fields. The same technique was used to obtain new solutions of the Einstein–Maxwell equations (Demianski and Newman, 1966; Demianski, 1972). In present paper, we will use this method to obtain a metric of a radiating rotating charged body.

The plan of the paper is as follows. In Section 2, starting from the metric of Bonner and Vaidya (1970) for a radiating charged body, we obtain by means of a complex coordinate transformation a new metric which describes the external gravitational field of a radiating rotating charged body. In Section 3 we study the properties of the metric. The classification of the Weyl tensor is then carried out using the spinor version (Pirani, 1964; Carmeli, 1974) of the Petrov classification. The Ricci tensor and the energy-momentum tensor are given. Section 4 is devoted to conclusions. Finally, for the sake of completeness and comparison, some calculated results are given in the Appendices.

## 2. THE METRIC OF A RADIATING ROTATING CHARGED BODY

The Vaidya–Bonner metric of a radiating charged body is as follows (Bonner and Vaidya, 1970):

$$g_{\mu\nu} = \begin{pmatrix} 1 - 2m/r + Q^2/r^2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{pmatrix} \tag{2.1}$$

where $\mu$ and $\nu$ run from 0 to 3 denoting $u$, $r$, $\theta$, $\varphi$, respectively. The quantities $m$ and $Q$ are arbitrary functions of $u$ and represent the mass and charge of the source, respectively.

In order to apply the "standard complex trick" (Newman and Janis, 1965a,b), we first write the metric (2.1) in the null tetrad form

$$g_{\mu\nu} = l_\mu n_\nu + n_\mu l_\nu - m_\mu \overline{m}_\nu - \overline{m}_\mu m_\nu \tag{2.2}$$

with

$$l_\mu = \delta^0_\mu, \qquad n_\mu = \frac{1}{2}\left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)\delta^0_\mu + \delta^1_\mu$$

$$m_\mu = -\frac{r}{\sqrt{2}}\,(\delta^2_\mu + i \sin\theta\, \delta^3_\mu) \tag{2.3}$$

Finding the contravariant null tetrad vectors corresponding to (2.3), and then complexifying them according to Newman and Janis (1965a,b), we have

$$l^\mu = \delta^\mu_1, \qquad n^\mu = \delta^\mu_0 - \frac{1}{2}\left[1 - m\left(\frac{1}{r} + \frac{1}{\bar{r}}\right) + \frac{Q^2}{r\bar{r}}\right]\delta^\mu_1$$

$$m^\mu = -\frac{1}{\sqrt{2}\bar{r}}\,(\delta^\mu_2 + i \sin\theta\, \delta^\mu_3) \tag{2.4}$$

where $r$ is allowed to take complex values and $\bar{r}$ is the complex conjugate of $r$.

Introduce the following complex coordinate transformations on vectors $l^\mu$, $n^\mu$, $m^\mu$, and $\overline{m}^\mu$ as used in Newman and Janis (1965a,b):

$$u' = u - ia\cos\theta, \qquad r' = r + ia\cos\theta \qquad (a = \text{const})$$

$$\theta' = \theta, \qquad \varphi' = \varphi \tag{2.5}$$

The corresponding transformation matrix for the 4-vector can be written as

$$a'^\mu_\nu = \begin{pmatrix} 1 & 0 & ia\sin\theta & 0 \\ 0 & 1 & -ia\sin\theta & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.6}$$

If now we restrict $r'$ and $u'$ to be real as in Newman and Janis (1965a,b) we obtain the following tetrad:

$$l'^\mu = \delta^\mu_1, \qquad n'^\mu = \delta^\mu_0 - \tfrac{1}{2}(1 - 2mr\rho\bar{\rho} + Q^2\rho\bar{\rho})\delta^\mu_1$$

$$m'^\mu = -\frac{\bar{\rho}}{\sqrt{2}}\left(ia\sin\theta\,\delta^\mu_0 - ia\sin\theta\,\delta^\mu_1 + \delta^\mu_2 + \frac{i}{\sin\theta}\,\delta^\mu_3\right) \tag{2.7}$$

where

$$\rho = -\frac{1}{r - ia\cos\theta} \tag{2.8}$$

Removing the prime and applying equation (2.2), we finally arrive at

$$g^{\mu\nu} = \begin{pmatrix} -a^2\sin^2\theta\,\rho\bar{\rho} & (a^2+r^2)\rho\bar{\rho} & 0 & -a\rho\bar{\rho} \\ (a^2+r^2)\rho\bar{\rho} & (2mr - Q^2 - r^2 - a^2)\rho\bar{\rho} & 0 & a\rho\bar{\rho} \\ 0 & 0 & -\rho\bar{\rho} & 0 \\ -a\rho\bar{\rho} & a\rho\bar{\rho} & 0 & -\rho\bar{\rho}/\sin^2\theta \end{pmatrix} \tag{2.9}$$

whereas the covariant of the metric is given by

$$g_{\mu\nu} = \begin{pmatrix} 1-(2mr-Q^2)\rho\bar{\rho} & 1 & 0 & a(2mr-Q^2)\rho\bar{\rho}\sin^2\theta \\ 1 & 0 & 0 & -a\sin^2\theta \\ 0 & 0 & -\dfrac{1}{\rho\bar{\rho}} & 0 \\ a(2mr-Q^2)\rho\bar{\rho}\sin^2\theta & -a\sin^2\theta & 0 & \sin^4\theta\left[(Q^2-2mr)a^2\rho\bar{\rho} - \dfrac{a^2+r^2}{\sin^2\theta}\right] \end{pmatrix} \tag{2.10}$$

This is the new space-time metric we obtain. It is a natural nonstationary generalization of the Kerr–Newman metric, where the latter has the same form, but with constants $m$ and $Q$. In metrics (2.9) and (2.10), $m(u)$ and $Q(u)$ are the mass and charge of the radiating rotating charged body as seen by an observer at infinity, respectively, and they are arbitrary functions of the retard time coordinate $u$. The total angular momentum of the body is given by $m(u)a$, where $a$ is a constant just as in the Kerr–Newman case.

## 3. THE PROPERTIES OF THE GRAVITATIONAL FIELD

In order to investigate the gravitational field (2.10) and compare the results with those of the Kerr–Newman metric and the nonstationary Kerr metric given in Carmeli (1982) and Carmeli and Kaye (1977, 1979), we use another null tetrad; the contravariant components of the null tetrad for the metric (2.10) are

$$\begin{aligned} l^\mu &= \delta_1^\mu \\ n^\mu &= \rho\bar{\rho}\left((r^2+a^2)\delta_0^\mu + \frac{2mr - Q^2 - (r^2+a^2)}{2}\,\delta_1^\mu + a\delta_3^\mu\right) \\ m^\mu &= -\frac{\bar{\rho}}{\sqrt{2}}\left(ia\sin\theta\,\delta_0^\mu + \delta_2^\mu + \frac{i}{\sin\theta}\,\delta_3^\mu\right) \end{aligned} \tag{3.1}$$

whereas the covariants of the null tetrad are given by

$$l_\mu = \delta_\mu^0 - a\sin^2\theta\,\delta_\mu^3$$

$$n_\mu = \rho\bar{\rho}\left(\frac{r^2 + a^2 + Q^2 - 2mr}{2}\delta_\mu^0 + \frac{\delta_\mu^1}{\rho\bar{\rho}} + \frac{2mr - Q^2 - (r^2 + a^2)}{2}a\sin^2\theta\,\delta_\mu^3\right)$$

$$m_\mu = -\frac{\bar{\rho}}{\sqrt{2}}\left(ia\sin\theta\,\delta_\mu^0 - \frac{\delta_\mu^2}{\rho\bar{\rho}} - i(r^2 + a^2)\sin\theta\,\delta_\mu^3\right) \tag{3.2}$$

The spin coefficients can now be calculated from the definitions (Carmeli, 1982; Carmeli and Kaye, 1977, 1979) and with aid of the Christoffel symbols which are listed in Appendix A, we find

$$\kappa = \epsilon = \sigma = \lambda = 0, \qquad \rho = -\frac{1}{r - ia\cos\theta}, \qquad \pi = \frac{ia\sin\theta\,\rho^2}{\sqrt{2}}$$

$$\beta = -\frac{\cot\theta\,\bar{\rho}}{2\sqrt{2}}, \qquad \alpha = \pi - \bar{\beta}, \qquad \mu = -\frac{2mr - Q^2 - (r^2 + a^2)}{2}\rho^2\bar{\rho}$$

$$\nu = -\frac{ia\sin\theta}{\sqrt{2}}\left(r\frac{dm}{du} - Q\frac{dQ}{du}\right)\rho^2\bar{\rho}, \qquad \gamma = \mu + \frac{(r - m)\rho\bar{\rho}}{2},$$

$$\tau = -\frac{ia\sin\theta\,\rho\bar{\rho}}{\sqrt{2}} \tag{3.3}$$

In the present case we note the presence of a spin coefficient, $\nu$, that is zero in the case of the Kerr–Newman metric. The rest of the spin coefficients are identical to those of the Kerr–Newman metric with variable mass and charge. The presence of this spin coefficient will in turn give rise to terms in the other Newman–Penrose quantities which do not appear in the Kerr–Newman case, as will be seen below.

The tetrad components of the trace-free Ricci tensor and Ricci scalar are calculated in Appendix B and are found to be

$$\Phi_{00} = \Phi_{01} = \Phi_{02} = 0, \qquad \Phi_{11} = \tfrac{1}{2}Q^2(\rho\bar{\rho})^2$$

and

$$\Phi_{12} = -\frac{ia\sin\theta}{\sqrt{2}}\left(\frac{\rho^2\bar{\rho}}{2}\frac{dm}{du} + Q(\rho\bar{\rho})^2\frac{dQ}{du}\right)$$

$$\Phi_{22} = -\frac{a^2\sin^2\theta\,(\rho\bar{\rho})^2}{2}\left[r\frac{d^2m}{du^2} - Q\frac{d^2Q}{du^2} - \left(\frac{dQ}{du}\right)^2\right]$$

$$- r(\rho\bar{\rho})^2\left(r\frac{dm}{du} - Q\frac{dQ}{du}\right)$$

$$R = -24\Lambda = 0 \tag{3.4}$$

The tetrad components of the Weyl tensor are (see Appendix B)

$$\psi_0 = \psi_1 = 0, \qquad \psi_2 = \left(\frac{m}{\bar{\rho}} + Q^2\right)\rho^3\bar{\rho}$$

$$\psi_3 = -i\,\frac{\rho^2\bar{\rho}}{2\sqrt{2}}\left[\frac{dm}{du} + 4\rho\left(r\,\frac{dm}{du} - Q\,\frac{dQ}{du}\right)\right]a \sin\theta$$

$$\psi_4 = \left\{\left[r\,\frac{d^2m}{du^2} - Q\,\frac{d^2Q}{du^2} - \left(\frac{dQ}{du}\right)^2\right]\frac{\rho^3\bar{\rho}}{2} + \left(r\,\frac{dm}{du} - Q\,\frac{dQ}{du}\right)\rho^4\bar{\rho}\right\}a^2 \sin^2\theta \tag{3.5}$$

We note the presence of $\Phi_{12}$, $\Phi_{22}$, $\psi_3$, and $\psi_4$ in (3.4) and (3.5), which are zero for the Kerr–Newman metric.

Since $\psi_0 = \psi_1 = 0$ and $3\psi_2\psi_4 \neq \psi_3^2$, we can show that the metric is algebraically special of Petrov type II with repeated principal null vector $l^\mu$. This is different from the Kerr–Newman metric, which is Petrov type D with two repeated principal null vectors $l^\mu$ and $n^\mu$.

The three optical scalars (shear, twist, and expansion) are

$$\sigma = [\tfrac{1}{2}l_{(\mu;\nu)}l^{\mu;\nu} - \Theta^2]^{1/2} = 0$$

$$\omega = [\tfrac{1}{2}l_{(\mu;\nu)}l^{\mu;\nu}]^{1/2} = -a\rho\bar{\rho}\cos\theta$$

$$\Theta = -\tfrac{1}{2}l^\mu_{;\mu} = -r\rho\bar{\rho} \tag{3.6}$$

Hence the metric contains a shear-free, twisting, and diverging null congruence identical to that of the Kerr–Newman metric.

We now calculate the components of the Ricci tensor by expanding it in terms of tetrad components (3.4). Since the Ricci scalar is zero, we have $R_{\mu\nu} = R_{mn}Z_\mu{}^m Z_\nu{}^n$, which is equal to the energy-momentum tensor $T_{\mu\nu}$. Hence, the energy-momentum tensor can be expressed as

$$\begin{aligned} T_{\mu\nu} = {} & 4Q^2(\rho\bar{\rho})^2[l_{(\mu}n_{\nu)} + m_{(\mu}\bar{m}_{\nu)}] \\ & - \left\{a^2 \sin^2\theta\,(\rho\bar{\rho})^2\left[r\,\frac{d^2m}{du^2} - Q\,\frac{d^2Q}{du^2} - \left(\frac{dQ}{du}\right)^2\right]\right. \\ & \left. + 2r\rho\bar{\rho}^2\left(r\,\frac{dm}{du} - Q\,\frac{dQ}{du}\right)\right\}l_\mu l_\nu - \frac{4a\sin\theta}{\sqrt{2}}\frac{dm}{du}\,\rho\bar{\rho}\,Im(l_{(\mu}\bar{m}_{\nu)}\rho) \\ & - \frac{8Q(\rho\bar{\rho})^2 a\sin\theta}{\sqrt{2}}\frac{dQ}{du}\,Im(l_{(\mu}\bar{m}_{\nu)}) \end{aligned} \tag{3.7}$$

Using equations (3.2) and (3.7), we can write directly the components of the

Ricci tensor and energy-momentum tensor. The energy-momentum tensor can be divided into three parts; the first one, $T^{(1)}_{\mu\nu} = 4Q^2(\rho\bar{\rho})^2[l_{(\mu}n_{\nu)} + m_{(\mu}\bar{m}_{\nu)}]$, is the energy-momentum tensor of the electromagnetic field, which is equal to that of the Kerr–Newman metric; the second part

$$T^{(2)}_{\mu\nu} = -2r\rho^2\bar{\rho}^2\left(r\frac{dm}{du} - Q\frac{dQ}{du}\right)l_\mu l_\nu$$

describes a Bonner–Vaidya-type radiative field; and the third one

$$T^{(3)}_{\mu\nu} = -\left\{a^2 \sin^2\theta\ (\rho\bar{\rho})^2\left[r\frac{d^2m}{du^2} - Q\frac{d^2Q}{du^2} - \left(\frac{dQ}{du}\right)^2\right]\right\}l_\mu l_\nu$$
$$- \frac{4a \sin\theta}{\sqrt{2}}\frac{dm}{du}\rho\bar{\rho}Im(l_{(\mu}\bar{m}_{\nu)}\rho) - \frac{8Q(\rho\bar{\rho})^2 a \sin\theta}{\sqrt{2}}\frac{dQ}{du}Im(l_{(\mu}\bar{m}_{\nu)})$$

is a residual contribution. Asymptotically, as $a/r$ tends to zero, one sees that the residual parts are of the order $r^{-3}$ and $r^{-4}$. Hence, due to the presence of the residual term in the energy-momentum tensor, only asymptotically does the nonstationary generalization of the Kerr–Newman metric behave like the Bonner–Vaidya metric.

## 4. CONCLUSION

In view of the above discussion, a new metric depending on three arbitrary parameters has been presented. The metric describes the gravitational field of a radiating rotating charged body. It is algebraically special of Petrov type II according to the classification of the Weyl tensor, with a twisting, shear-free, null congruence identical to that of the Kerr–Newman metric.

## APPENDIX A. CHRISTOFFEL SYMBOLS FOR THE METRIC OF A RADIATING ROTATING CHARGED BODY

The nonzero Christoffel symbols of the second kind for the gravitational field of the radiating rotating charged body described by equations (2.9) and (2.10) are given by

$$\Gamma^\mu_{\alpha\beta} = g^{\mu\gamma}\Gamma_{\gamma\alpha\beta}$$

$$\Gamma^0_{00} = -\left(\frac{dm}{dr}r - Q\frac{dQ}{dt}\right)a^2 \sin^2\theta\ \rho^2\bar{\rho}^2 - (mr^2 - Q^2r)\rho^2\bar{\rho}^2 + ma^2\rho^2\bar{\rho}^2$$
$$- (2mr - Q^2)ra^2 \sin^2\theta\ \rho^3\bar{\rho}^3$$

$$\Gamma^0_{02} = -(2mr - Q^2)a^2 \sin\theta \cos\theta\, \rho^2\bar{\rho}^2$$

$$\Gamma^0_{03} = (2mr^2 - Q^2r)a^3 \sin^4\theta\, \rho^3\bar{\rho}^3 + (2mr - Q^2)ar \sin^2\theta\, \rho^2\bar{\rho}^2 - ma^3 \sin^4\theta\, \rho^2\bar{\rho}^2$$
$$+ \left(\frac{dm}{dt} r - Q \frac{dQ}{dt}\right)a^3 \sin^4\theta\, \rho^2\bar{\rho}^2 - ma \sin^2\theta\, \rho\bar{\rho}$$

$$\Gamma^0_{12} = a^2 \sin\theta \cos\theta\, \rho\bar{\rho}$$

$$\Gamma^0_{13} = ra \sin^2\theta\, \rho\bar{\rho}$$

$$\Gamma^0_{22} = r(a^2 + r^2)\rho\bar{\rho}$$

$$\Gamma^0_{23} = (2mr - Q^2)a^3 \sin^3\theta \cos\theta\, \rho^2\bar{\rho}^2$$

$$\Gamma^0_{33} = -(mr^4 - Q^2r^3 - Q^2ra^2)a^2 \sin^4\theta\, \rho^3\bar{\rho}^3$$
$$- mr^2a^4 \sin^6\theta\, \rho^3\bar{\rho}^3 + ma^6 \sin^4\theta \cos^2\theta\, \rho^3\bar{\rho}^3$$
$$- \left(\frac{dm}{dt} r - Q \frac{dQ}{dt}\right)a^4 \sin^6\theta\, \rho^2\bar{\rho}^2 + ra^2 \sin^4\theta\, \rho\bar{\rho} + r \sin^2\theta$$

$$\Gamma^1_{00} = (2mr^2 - Q^2r)a^2 \sin^2\theta\, \rho^3\bar{\rho}^3 - (2mr - Q^2)^2 r\rho^3\bar{\rho}^3 + \left(\frac{dm}{dt} r - Q \frac{dQ}{dt}\right)a^2$$
$$\times \sin^2\theta\, \rho^2\bar{\rho}^2 + (2mr - Q^2)m\rho^2\bar{\rho}^2 + (2mr^2 - Q^2r)\rho^2\bar{\rho}^2 - ma^2 \sin^2\theta\, \rho^2\bar{\rho}^2$$
$$- \left(\frac{dm}{dt} r - Q \frac{dQ}{dt}\right)\rho\bar{\rho} - m\rho\bar{\rho}$$

$$\Gamma^1_{01} = (2mr - Q^2)r\rho^2\bar{\rho}^2 - m\rho\bar{\rho}$$

$$\Gamma^1_{03} = (2mr - Q^2)^2 ra \sin^2\theta\, \rho^3\bar{\rho}^3 - (2mr - Q^2)ra^3 \sin^4\theta\, \rho^3\bar{\rho}^3$$
$$- (2mr - Q^2)ma \sin^2\theta\, \rho^2\bar{\rho}^2 - (2mr - Q^2)ra \sin^2\theta\, \rho^2\bar{\rho}^2 + ma^3 \sin^4\theta\, \rho^2\bar{\rho}^2$$
$$- \left(\frac{dm}{dt} r - Q \frac{dQ}{dt}\right)a^3 \sin^4\theta\, \rho^2\bar{\rho}^2 + ma \sin^2\theta\, \rho\bar{\rho}$$

$$\Gamma^1_{12} = -a^2 \sin\theta \cos\theta\, \rho\bar{\rho}$$
$$\Gamma^1_{13} = -(2mr - Q^2)ra \sin^2\theta\, \rho^2\bar{\rho}^2 + ma \sin^2\theta\, \rho\bar{\rho} - ra \sin^2\theta\, \rho\bar{\rho}$$
$$\Gamma^1_{22} = (2mr - Q^2)r\rho\bar{\rho} - ra^2 \sin^2\theta\, \rho\bar{\rho} - r$$
$$\Gamma^1_{33} = -(2mr - Q^2)^2 ra^2 \sin^4\theta\, \rho^3\bar{\rho}^3 + (2mr - Q^2)ra^4 \sin^6\theta\, \rho^3\bar{\rho}^3 + \left(\frac{dm}{dt} r - Q \frac{dQ}{dt}\right)$$
$$\times a^4 \sin^6\theta\, \rho^2\bar{\rho}^2 + (2mr - Q^2)ra^2 \sin^4\theta\, \rho^2\bar{\rho}^2 + (2mr - Q^2)ma^2 \sin^4\theta\, \rho^2\bar{\rho}^2$$
$$- ma^4 \sin^6\theta\, \rho^2\bar{\rho}^2 + \left(\frac{dm}{dt} r - Q \frac{dQ}{dt}\right)a^2 \sin^4\theta\, \rho\bar{\rho} - ma^2 \sin^4\theta\, \rho\bar{\rho} + (2mr - Q^2)$$

$$\times\, r \sin^2\theta\, \rho\bar\rho - ra^2 \sin^4\theta\, \rho\bar\rho - r\sin^2\theta$$

$$\Gamma^2_{00} = -(2mr - Q^2)a^2 \sin\theta \cos\theta\, \rho^3\bar\rho^3$$

$$\Gamma^2_{03} = (2mr - Q^2)r^2 a \sin\theta\cos\theta\, \rho^3\bar\rho^3 + (2mr - Q^2)a^3 \sin\theta\cos\theta\, \rho^3\bar\rho^3$$

$$\Gamma^2_{12} = r\rho\bar\rho$$

$$\Gamma^2_{13} = -a \sin\theta\cos\theta\, \rho\bar\rho$$

$$\Gamma^2_{22} = -a^2 \sin\theta\cos\theta\, \rho\bar\rho$$

$$\Gamma^2_{33} = -(2mr - Q^2)a^4 \sin^5\theta\cos\theta\, \rho^3\bar\rho^3 - 2(2mr - Q^2)a^2 \sin^3\theta\cos\theta\, \rho^2\bar\rho^2 - (r^2 + a^2)\sin\theta\cos\theta\, \rho\bar\rho$$

$$\Gamma^3_{00} = -\left(\frac{dm}{dt} r - Q\frac{dQ}{dt}\right)a\rho^2\bar\rho^2 + ma\rho^2\bar\rho^2 - (2mr - Q^2)ra\rho^3\bar\rho^3$$

$$\Gamma^3_{02} = -(2mr - Q^2)a \cos\theta\, \mathrm{cosec}\,\theta\, \rho^2\bar\rho^2$$

$$\Gamma^3_{03} = (2mr - Q^2)ra^2 \sin^2\theta\, \rho^3\bar\rho^3 - ma^2\sin^2\theta\, \rho^2\bar\rho^2 + \left(\frac{dm}{dt} r - Q\frac{dQ}{dt}\right)a^2 \sin^2\theta\, \rho^2\bar\rho^2$$

$$\Gamma^3_{12} = a\cos\theta\, \mathrm{cosec}\,\theta\, \rho\bar\rho$$

$$\Gamma^3_{13} = r\rho\bar\rho$$

$$\Gamma^3_{22} = ra\rho\bar\rho$$

$$\Gamma^3_{23} = (2mr - Q^2)a^2 \sin\theta\cos\theta\, \rho^2\bar\rho^2 + \cos\theta\, \mathrm{cosec}\,\theta$$

$$\Gamma^3_{33} = -(2mr - Q^2)ra^3 \sin^4\theta\, \rho^3\bar\rho^3 - \left(\frac{dm}{dt} r - Q\frac{dQ}{dt}\right)a^3 \sin^4\theta\, \rho^2\bar\rho^2 + ma^3 \sin^4\theta\, \rho^2\bar\rho^2 + ra\sin^2\theta\, \rho\bar\rho \qquad \text{(A1)}$$

## APPENDIX B. TETRAD COMPONENTS OF THE RICCI TENSOR, RICCI SCALAR, AND WEYL TENSOR

The tetrad components of the Ricci tensor $\Phi_{00}$, $\Phi_{01}$, $\Phi_{02}$, $\Phi_{11}$, $\Phi_{12}$, and $\Phi_{22}$ are calculated using the Newman–Penrose equations:

$$D\rho - \bar\delta\kappa = (\rho^2 + \sigma\bar\sigma) + (\epsilon + \bar\epsilon)\rho - \bar\kappa\tau - \kappa(3\alpha + \bar\beta - \pi) + \Phi_{00}$$

$$D\alpha - \bar\delta\epsilon = (\rho + \bar\epsilon - 2\epsilon)\alpha + \beta\bar\sigma - \bar\beta\epsilon - \kappa\lambda - \bar\kappa\gamma + (\epsilon + \rho)\pi + \Phi_{10}$$

$$D\lambda - \bar\delta\pi = (\rho\lambda + \bar\sigma\mu) + \pi^2 + (\alpha - \bar\beta)\pi - \nu\bar\kappa - (3\epsilon - \bar\epsilon)\lambda + \Phi_{20}$$

$$D\gamma - \Delta\epsilon + \delta\alpha - \bar\delta\beta = (\tau + \bar\pi)\alpha + (\bar\tau + \pi)\beta - (\epsilon + \bar\epsilon)\gamma - (\gamma + \bar\gamma)\epsilon + \tau\pi - \nu\kappa + (\mu\rho - \lambda\sigma) + \alpha\bar\alpha + \beta\bar\beta - 2\alpha\beta + (\rho - \bar\rho)\gamma + (\mu - \bar\mu)\epsilon + 2\Phi_{11}$$

$$\delta\gamma - \Delta\beta = (\tau - \bar\alpha - \beta)\gamma + \mu\tau - \sigma\nu - \epsilon\bar\nu - (\gamma - \bar\gamma - \mu)\beta + \alpha\bar\lambda + \Phi_{12}$$

$$\delta\nu - \Delta\mu = (\mu^2 + \lambda\bar\lambda) + (\gamma + \bar\gamma)\mu - \bar\nu\pi + (\tau - 3\beta - \bar\alpha)\nu + \Phi_{22} \qquad \text{(B1)}$$

The Ricci scalar $R = -24\Lambda$ can be calculated from

$$D\mu - \delta\pi + \delta\alpha - \bar{\delta}\beta = (\bar{\rho} + \rho)\mu + (\rho - \bar{\rho})\gamma - (\bar{\alpha} - \beta)\pi - (\epsilon\bar{\mu} + \bar{\epsilon}\mu) - \nu\kappa + \alpha\bar{\alpha} + \beta\bar{\beta} + \pi\bar{\pi} - 2\alpha\beta + \Phi_{11} + 3\Lambda \tag{B2}$$

In order to calculate the tetrad components of the Weyl tensor $\psi_0$, $\psi_1$, $\psi_2$, $\psi_3$, and $\psi_4$, we can use the following Newman–Penrose equations:

$$\begin{aligned}
D\sigma - \delta\kappa &= (\rho + \bar{\rho})\sigma + (3\epsilon - \bar{\epsilon})\sigma - \kappa(\tau - \bar{\pi} + \bar{\alpha} + 3\beta) + \psi_0 \\
D\beta - \delta\epsilon &= (\alpha + \pi)\sigma + (\bar{\rho} - \bar{\epsilon})\beta - (\mu + \gamma)\kappa - (\bar{\alpha} - \bar{\pi})\epsilon + \psi_1 \\
\Delta\rho - \bar{\delta}\tau &= -(\rho\bar{\mu} + \sigma\lambda) + (\bar{\beta} - \alpha - \bar{\tau})\tau + (\gamma + \bar{\gamma})\rho + \nu\kappa - \psi_2 - 2\Lambda \\
\Delta\alpha - \bar{\delta}\gamma &= (\rho + \epsilon)\nu - (\tau + \beta)\lambda + (\bar{\gamma} - \bar{\mu})\alpha + (\bar{\beta} - \bar{\tau})\gamma - \psi_3 \\
\Delta\lambda - \bar{\delta}\nu &= -(\mu + \bar{\mu})\lambda - (3\gamma - \bar{\gamma})\lambda + (3\alpha + \bar{\beta} + \pi - \bar{\tau})\nu - \psi_4
\end{aligned} \tag{B3}$$

From the definitions of the intrinsic derivatives and the expressions for the null tetrad (3.1) we find that

$$\begin{aligned}
D &= \partial/\partial r \\
\Delta &= \rho\bar{\rho}[(r^2 + a^2)(\partial/\partial u) - \tfrac{1}{2}(r^2 + a^2 - 2mr + Q^2)(\partial/\partial r) + a(\partial/\partial\varphi)] \\
\delta &= -(\bar{\rho}/\sqrt{2})[ia \sin\theta\, (\partial/\partial u) + (\partial/\partial\theta) + i \operatorname{cosec}\theta\, (\partial/\partial\varphi)]
\end{aligned} \tag{B4}$$

Substituting the explicit expressions for the intrinsic derivatives and the spin coefficients as given in (B4) and (3.3), respectively, into (B1)–(B3), we find

$$\begin{aligned}
\Phi_{00} &= \Phi_{01} = \Phi_{02} = 0 \\
\Phi_{11} &= \frac{Q^2}{2}(\rho\bar{\rho})^2 \\
\Phi_{12} &= -\frac{ia \sin\theta}{\sqrt{2}}\left(\frac{\rho^2\bar{\rho}}{2}\frac{dm}{du} + Q(\rho\bar{\rho})^2\frac{dQ}{du}\right) \\
\Phi_{22} &= -\frac{a^2 \sin^2\theta\, (\rho\bar{\rho})^2}{2}\left[r\frac{d^2m}{du^2} - Q\frac{d^2Q}{du^2} - \left(\frac{dQ}{du}\right)^2\right] - r(\rho\bar{\rho})^2\left(r\frac{dm}{du} - Q\frac{dQ}{du}\right)
\end{aligned} \tag{B5}$$

$$\Lambda = 0 \tag{B6}$$

$$\begin{aligned}
\psi_0 &= \psi_1 = 0 \\
\psi_2 &= \left(\frac{m}{\bar{\rho}} + Q^2\right)\rho^3\bar{\rho}
\end{aligned}$$

$$\psi_3 = -i\,\frac{\rho^2\bar{\rho}}{2\sqrt{2}}\left[\frac{dm}{du} + 4\rho\left(r\,\frac{dm}{du} - Q\,\frac{dQ}{du}\right)\right]a \sin\theta$$

$$\psi_4 = \left\{\left[r\,\frac{d^2m}{du^2} - Q\,\frac{d^2Q}{du^2} - \left(\frac{dQ}{du}\right)^2\right]\frac{\rho^3\bar{\rho}}{2} + \left(r\,\frac{dm}{du} - Q\,\frac{dQ}{du}\right)\rho^4\bar{\rho}\right\}a^2 \sin^2\theta \qquad \text{(B7)}$$

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